

Tetraquarks, hadronic molecules, meson-meson scattering, and disconnected contributions in lattice QCD

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There are generally two types of Wick contractions in lattice QCD calculations of a correlation function: connected and disconnected ones. The disconnected contribution is difficult to calculate and noisy, thus it is often neglected. In the context of studying tetraquarks, hadronic molecules, and meson-meson scattering, we show that whenever there are both connected and singly disconnected contractions, the singly disconnected part gives the leading order contribution, and thus should never be neglected. As an explicit example, we show that information about the scalar mesons σ , $f_0(980)$, $a_0(980)$, and κ will be lost when neglecting the disconnected contributions.

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It is generally believed that quantum chromodynamics (QCD), the fundamental theory of strong interactions, allows for the existence of tetraquark states in addition to the normal mesons made of a quark and an antiquark. A tetraquark consists of two quarks and two antiquarks. Furthermore, if the interaction between two hadrons is strong enough, it should be possible that the hadrons can bind together to form a hadronic molecule. Even if the attraction is too weak to obtain a bound state, it might still be possible that the interaction is resonant. In this case, the resonance generated by the interaction is still called a hadronic molecule. Candidates of the tetraquark states and/or hadronic molecules among the light mesons include the light scalar mesons below 1 GeV, specifically the $f_0(500)$ (or σ), $f_0(980)$, $a_0(980)$, and $K_0^*(800)$ (or κ), see e.g. Refs. [1–4] and references therein. In the last decade, due to the continuing worldwide experimental efforts, quite a few prominent candidates of tetraquarks and hadronic molecules were discovered in the heavy flavor sector. Arguably the most intriguing ones are the $X(3872)$ [5], whose mass is extremely close to the $D^0\bar{D}^{*0}$ threshold, and the charged heavy quarkoniumlike states $Z_b(10610, 10650)^\pm$ [6] and the $Z_c(3900)^\pm$ [7,8].

Being intrinsically nonperturbative, lattice QCD is the ideal approach to study hadron spectroscopy including tetraquarks and hadronic molecules. For studying tetraquark states in lattice QCD, one calculates the two-point correlation function of an interpolating field containing four quarks. The basic form of such an interpolating field reads

$$\mathcal{O}_{ABCD}^{ij}(x) = [\bar{q}_A(x)\Gamma^i q_B(x)][\bar{q}_C(x)\Gamma^j q_D(x)], \quad (1)$$

where the capital indices denote the quark flavor, and Γ^i are the spin matrices. Notice that each pair of quark and antiquark is colorless. This can be done because Fierz transformation allows any gauge-invariant four-quark operator to be written in terms of a linear combination of products of two colorless quark-antiquark operators. The interpolating field used in a lattice calculation can be a linear combination of the one given in Eq. (1) with different spin and flavor structures. Depending on the flavor content, in general there are four types of Wick contractions in the calculation of the two-point correlation function. They are shown in Fig. 1.

In lattice calculations, Figs. 1(a) and 1(b) are called connected diagrams, and Figs. 1(c) and 1(d) are the disconnected ones. The disconnected contribution involves all-to-all propagators and is notoriously difficult to calculate and often noisy. So far, there have been quite a few lattice calculations by different groups on tetraquark states. The simulations performed before 2009 used the quenched approximation, see, e.g., the calculations of the light scalar mesons in Refs. [9–13]. All of them neglect the disconnected diagrams. Recently, dynamical calculations in lattice QCD of scalar tetraquarks appeared [14,15], where the disconnected diagrams were also neglected despite the fact that unitarity was broken by such a procedure. Yet, with the help of various new algorithms and the development of computing techniques, e.g. the distillation method [16], there have been lattice calculations including disconnected Wick contractions in the context of tetraquarks and meson-meson scattering, see, for instance, Refs. [17–21]. Energy levels above the meson-meson thresholds, which may be interpreted as the σ and the κ , were reported in Ref. [14]. It was shown later in a dynamical lattice calculation of $K\pi$

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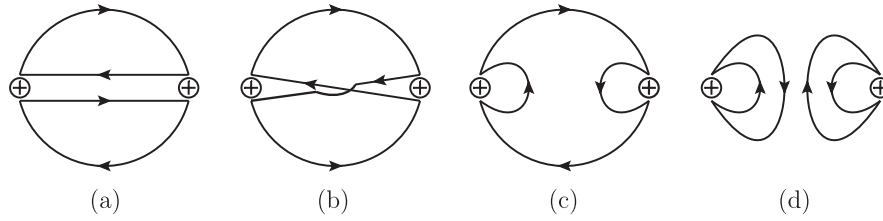


FIG. 1. Four different types of Wick contractions of the two-point correlation function in case of a four-quark interpolating field. The insertion of the interpolating field is denoted by \oplus . (a) and (b) are connected, (c) is singly disconnected, and (d) is doubly disconnected.

scattering [19] that the energy level above and near the $K\pi$ threshold is an artifact of neglecting the disconnected contributions. However, using similar lattice setups as those of Ref. [15] and also neglecting the disconnected diagrams, the ETM Collaboration investigated the isospin 1 and 1/2 channels with a few different four-quark interpolating fields. They did not find any state in addition to the scattering states in both channels. In view that the lattice calculations of Ref. [19] already showed the importance of the disconnected contributions, it would be helpful to analyze the problem theoretically from general principles. As will be shown in this paper using the large- N counting, with N being the number of colors, and chiral expansion, as long as there are both connected and disconnected contributions, the disconnected part is at least as important as the the connected one, or even more important depending on the flavor content of the interpolating field. Our conclusion can be applied when the correlator in question is a two-point correlation function of a four-quark interpolating field. This includes studies of tetraquarks, hadronic molecules, and meson-meson scattering observables.

To start, consider the large- N counting of the various types of contractions, and then present quantitative results on the impact of the disconnected diagrams in the sector of the light scalar mesons using unitarized chiral perturbation theory (CHPT).

When the number of colors is large, many interesting qualitative results can be obtained [22,23]. We will apply the N counting rules here to the Wick contractions. Notice that we do not need to assume tetraquarks to survive in the large- N limit, which was done in Refs. [24,25]. What we need is just an N counting scheme which is assumed to work for $N = 3$ as well. In this way, tetraquarks, hadronic molecules, and meson-meson scattering can be discussed in a unified way with the interpolating field given in Eq. (1).¹

¹Hadronic molecules made of two mesons cannot exist when N is infinite even if an infinite chain of $1/N$ suppressed scattering amplitudes is summed up. This is because the loop expansion of the meson-meson scattering coincides with the $1/N$ expansion. An infinite summation may be roughly represented by $1/(N - f)$, where f is an N -independent factor. Apparently, if N is large enough, this summation cannot develop a pole. For the behavior of the scalar mesons at a large but not infinite N , we refer to the review [26] and references therein.

Let us analyze the power of N of the different contractions in Fig. 1. Both the cross connected diagram 1(b) and the singly disconnected diagram 1(c) have only one quark loop, thus they are of order N . The direct connected diagram 1(a) and the doubly disconnected diagram 1(d) have two quark loops. However, at least two gluons need to be exchanged to connect the two quark loops, since otherwise there is no interaction between the two parts and a pole can never emerge. Because the quark-gluon vertex goes as $1/\sqrt{N}$, diagrams 1(a) and 1(d) should be of order $N^2/(\sqrt{N})^4 = N^0$. Therefore, the direct connected and doubly disconnected diagrams are suppressed by $1/N$ in comparison with the cross connected and singly disconnected ones.

We divide the four-quark interpolating fields into different types²:

- (i) Those with one quark having the same flavor as one antiquark, i.e., $\bar{q}_A q_B \bar{q}_B q_C$. The possible Wick contractions are given by diagrams 1(a) and 1(c) [if $q_C = q_A$, then diagram 1(d) also contributes]. The leading order contribution comes from the singly disconnected diagram 1(c).
- (ii) Those with two same quarks (or antiquarks), i.e., $\bar{q}_A q_B \bar{q}_C q_B$. The contractions include 1(a) and 1(b), both of which are connected, and diagram 1(b) dominates. If $q_C = q_B$, the singly disconnected diagram 1(c) will also contribute. It is of the same order as 1(b), and it is thus not legitimate to neglect it.
- (iii) Those with four different flavors. The two-point correlator only has a contraction of diagram 1(a) which is connected.

From the above large- N counting, we can conclude that as long as the singly disconnected contraction contributes, it is of the leading order. Neglecting such a contribution will definitely lead to unphysical results which are difficult to be related to the real world.

For the light pseudoscalar meson scattering, which is under intense investigation in lattice QCD currently (see e.g. Refs. [19–21,27,28] and references therein), the same conclusion can be obtained without the large- N arguments. In the low-energy regime of QCD, the lowest-lying pseudoscalar mesons π , K , and η can be treated as the

²Since we focus on the correlators with interpolating fields of four quarks, we will not discuss another type of tetraquark which is excited inside a $\bar{q}q$ meson as proposed in Ref. [25].

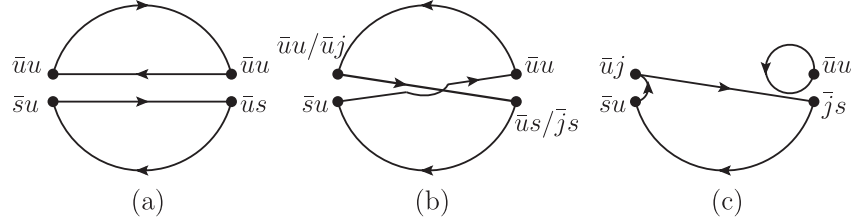


FIG. 2. The connected Wick contractions for the process $\eta_u K^+ \rightarrow \eta_u K^+$ are given by (a) and (b), while (b) and (c) exhaust all possible contractions for the $\phi_{j\bar{u}} K^+ \rightarrow \eta_u \phi_{j\bar{s}}$.

(pseudo-) Goldstone bosons of the spontaneous chiral symmetry breaking, and their interactions among themselves and with other particles can be described by CHPT [29,30]. The leading order Lagrangian of CHPT only has a single flavor trace. Noticing that the number of circles in the Wick contractions is in line with the number of flavor traces, only diagrams 1(b) and 1(c) can appear at leading order. Therefore, we again arrive at the conclusion that the singly disconnected diagram is of leading order, whenever it contributes, and cannot be neglected. As an explicit example, we will study the light scalar mesons below 1 GeV using the chiral unitary approach.

In the chiral unitary approach as developed in Ref. [31], the meson-meson scattering amplitudes derived from the leading order CHPT are put in the kernel of the Bethe-Salpeter equation, which is reduced from an integral equation to an algebraic equation due to an on-shell approximation. The equation reads as

$$T(s) = V(s)[1 - G(s)V(s)]^{-1}, \quad (2)$$

where $V(s)$ is the scattering amplitudes projected to the S -wave, and $G(s)$ is the normal scalar two-point loop function. For coupled channels, $V(s)$, $G(s)$, and $T(s)$ are matrix valued. With one parameter, which is a three-momentum sharp cutoff $q_{\max} \simeq 0.9$ GeV used to regularize the divergent scalar loop integral, the scattering data in the isospin $I = 0$ and 1 channels are well described.³ More interestingly, the lowest-lying scalar mesons σ [also called $f_0(500)$] and $f_0(980)$ in the isoscalar channel and the $a_0(980)$ in the isovector channel can be generated at the right positions. We will show that if the disconnected contributions are discarded from the scattering amplitudes, the poles of the scalar mesons will be moved far away from their physical values. Although extensions of this approach to higher orders exist in the literature [32–35], it is enough to use the simplest formalism in Ref. [31] for our purpose.

As a first step, one has to separate the disconnected (or connected) contributions from the full scattering amplitudes. At leading order in CHPT, i.e. $\mathcal{O}(p^2)$, with p

representing a small momentum or a Goldstone boson mass, the separation of the different Wick contractions can be done by merely extending the normal SU(3) CHPT to SU(4) with one auxiliary quark flavor,⁴ to be denoted by j . The meson fields are collected in a 4×4 matrix

$$\Phi = \begin{pmatrix} \eta_u & \pi^+ & K^+ & \phi_{u\bar{j}} \\ \pi^- & \eta_d & K^0 & \phi_{d\bar{j}} \\ K^- & \bar{K}^0 & \eta_s & \phi_{s\bar{j}} \\ \phi_{j\bar{u}} & \phi_{j\bar{d}} & \phi_{j\bar{s}} & \eta_j \end{pmatrix}, \quad (3)$$

where $\eta_f = \bar{f}f$ ($f = u, d, s, j$), $\pi^0 = (\eta_u - \eta_d)/\sqrt{2}$, and $\eta = (\eta_u + \eta_d - 2\eta_s)/\sqrt{6}$. We will explain the procedure by an example. Let us calculate the connected contribution to the leading order scattering amplitude of the process $\pi^0 K^+ \rightarrow \pi^0 K^+$, which may be written as

$$V_{\pi^0 K^+ \rightarrow \pi^0 K^+} = \frac{1}{2}(V_{\eta_u K^+ \rightarrow \eta_u K^+} + V_{\eta_d K^+ \rightarrow \eta_d K^+} - 2V_{\eta_u K^+ \rightarrow \eta_d K^+}). \quad (4)$$

Because the flavor content of the K^+ is $\bar{s}u$, it is obvious that the last term in the above equation only has disconnected Wick contractions due to the presence of one η_d . The second term has both connected and disconnected contractions. However, the d and \bar{d} quarks have to be contracted within the two η_d 's, so that the connected contraction corresponds to a double trace in the flavor space. Such a contribution vanishes at leading order of CHPT as discussed above. There are two different connected contractions for the $\eta_u K^+ \rightarrow \eta_u K^+$, as shown in Figs. 2(a) and 2(b). Diagram 2(a) does not contribute at the leading order. In addition, this process has disconnected contractions including the singly disconnected one. The cross connected diagram 2(b) can be picked out by substituting the auxiliary quark j for one cross line of the u quark. To this end, one calculates the scattering amplitude of the reaction $\phi_{j\bar{u}} K^+ \rightarrow \eta_u \phi_{j\bar{s}}$. The auxiliary process

³Note, however, that better regularization methods are by now available. Here, since we are only interested in quantitative estimates, we continue using a sharp cutoff.

⁴At higher orders, one needs to use partially quenched CHPT [36,37] (for reviews, see Refs. [38,39]), and examples may be found in Refs. [38,40–42].

TABLE I. A comparison of the connected and full meson-meson scattering amplitudes denoted by $V^{(2),C}(s, t, u)$ and $V^{(2)}(s, t, u)$, respectively, at $\mathcal{O}(p^2)$.

Isospin	Processes	$V^{(2),C}(s, t, u)$	$V^{(2)}(s, t, u)$
$I = 0$	$\pi\pi \rightarrow \pi\pi$	$-\frac{1}{2F^2}(s - 2M_\pi^2)$	$-\frac{1}{F^2}(2s - M_\pi^2)$
	$K\bar{K} \rightarrow K\bar{K}$	0	$-\frac{3}{2F^2}(s + t - 2M_K^2)$
	$K\bar{K} \rightarrow \pi\pi$	0	$\frac{\sqrt{6}s}{4F^2}$
$I = 1$	$\pi\eta \rightarrow \pi\eta$	$\frac{1}{18F^2}(3s - 4M_\pi^2 - 2M_\eta^2)$	$-\frac{M_\pi^2}{3F^2}$
	$K\bar{K} \rightarrow K\bar{K}$	0	$-\frac{1}{2F^2}(s + t - 2M_K^2)$
	$K\bar{K} \rightarrow \pi\eta$	$-\frac{\sqrt{6}}{18F^2}(3s - M_\pi^2 - M_\eta^2 - 4M_K^2)$	$-\frac{\sqrt{6}}{36F^2}(9s - M_\pi^2 - 3M_\eta^2 - 8M_K^2)$
$I = \frac{1}{2}$	$\pi K \rightarrow \pi K$	$-\frac{1}{4F^2}(s - M_\pi^2 - M_K^2)$	$-\frac{1}{4F^2}(4s + 3t - 4M_\pi^2 - 4M_K^2)$

$\phi_{j\bar{u}}K^+ \rightarrow \eta_u\phi_{j\bar{s}}$ has two Wick contractions as shown in Figs. 2(b) and 2(c). However, diagram 2(c) vanishes at leading order due to a double trace again. The mass of the j quark is required to be the same as that of the replaced u quark, and this guarantees that the amplitude corresponds to the desired Wick contraction. Hence, we obtain

$$\begin{aligned}
 V_{\pi^0 K^+ \rightarrow \pi^0 K^+}^{(2),C}(s, t, u) &= \frac{1}{2} V_{\phi_{j\bar{u}}K^+ \rightarrow \eta_u\phi_{j\bar{s}}}^{(2)}(s, t, u)|_{m_j=m_u} \\
 &= \frac{1}{4F^2}(s - M_\pi^2 - M_K^2), \quad (5)
 \end{aligned}$$

where F is the pion decay constant, $M_{\pi(K)}$ are the masses of the $\pi(K)$, and s, t, u are the conventional Mandelstam variables. Here, the superscript “(2)” denotes the order $\mathcal{O}(p^2)$, and “C” represents connected. The connected part in Eq. (5) is clearly different from the full leading order amplitude of $\pi^0 K^+ \rightarrow \pi^0 K^+$, which is given by $-t/(4F^2)$, and the difference is not negligible. For a comparison, both the connected and full scattering amplitudes at $\mathcal{O}(p^2)$ for seven most relevant processes are given in Table I.

Replacing $V(s)$ in Eq. (2) by the scattering amplitudes in Table I after the S -wave projection (an additional factor of $1/2$ should be multiplied to the $\pi\pi \rightarrow \pi\pi$ amplitude to account for the identical particles), one can search for poles of the T matrix. When the full amplitudes are used, with the three-momentum cutoff $q_{\max} = 0.9$ GeV, the pole of the σ is located at $(0.47 - i0.20)$ GeV in the second Riemann sheet [31]. If the disconnected contributions are neglected, the pole will move to $(0.84 - i0.62)$ GeV. It is moved deep in the complex plane and has nothing to do with the physical σ pole. Taking into account only the connected contractions, the $K\bar{K}$ channel decouples in the isoscalar case, and the $f_0(980)$ pole cannot be generated anymore. Similar to the case of the σ , when the disconnected contributions are dropped, both the $a_0(980)$ and κ poles will be moved far away from their physical values, $(1.07 - i0.34)$ GeV (in the third Riemann sheet) and $(0.93 - i0.54)$ GeV (in the second Riemann sheet), respectively. These values should be compared to

$(1.01 - i0.06)$ GeV [31] and $(0.73 - i0.25)$ GeV [43] (both in the second Riemann sheet) which are obtained when the full $\mathcal{O}(p^2)$ amplitudes are considered (note that $q_{\max} = 0.51$ GeV given in Ref. [43] is used for πK scattering).

In summary, we have discussed the contribution of the disconnected Wick contractions to the two-point correlation functions of the four-quark interpolating fields. The discussion applies to tetraquarks, hadronic molecules, and meson-meson scattering observables. Using the large- N counting of QCD, we have shown that the singly disconnected Wick contraction is always of the leading order whenever it contributes. This conclusion is further strengthened for the case of the light scalar mesons and the scattering processes involving pseudoscalar mesons—the singly disconnected contraction appears at the leading order of CHPT. The connected contribution to the leading order meson-meson scattering amplitudes is worked out by introducing an auxiliary quark flavor. Using unitarized CHPT, we have shown explicitly that the poles of the light scalar mesons move far away from their physical positions when neglecting the disconnected diagrams. The finding from an explicit lattice calculation in Ref. [19] that the disconnected contractions are important in the case of the $I = 1/2 K\pi$ scattering is in line with our conclusion. Therefore, in the lattice study of tetraquarks, hadronic molecules, and meson-meson scattering observables, whenever there are both connected and singly disconnected contributions, the disconnected part should never be dropped.

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- [1] R. L. Jaffe, *Phys. Rev. D* **15**, 267 (1977); **15**, 281 (1977).
- [2] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, *Phys. Rev. Lett.* **93**, 212002 (2004).
- [3] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, and A. E. Kudryavtsev, *Phys. Lett. B* **586**, 53 (2004).
- [4] C. Amsler and N. A. Törnqvist, *Phys. Rep.* **389**, 61 (2004).
- [5] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
- [6] A. Bondar *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **108**, 122001 (2012).
- [7] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **110**, 252001 (2013).
- [8] Z. Q. Liu *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **110**, 252002 (2013).
- [9] M. G. Alford and R. L. Jaffe, *Nucl. Phys.* **B578**, 367 (2000).
- [10] N. Mathur, A. Alexandru, Y. Chen, S. J. Dong, T. Draper, I. Horvath, F. X. Lee, K. F. Liu, S. Tamhankar, and J. Zhang, *Phys. Rev. D* **76**, 114505 (2007).
- [11] H. Suganuma, K. Tsumura, N. Ishii, and F. Okiharu, *Prog. Theor. Phys. Suppl.* **168**, 168 (2007).
- [12] M. Loan, Z.-H. Luo, and Y. Y. Lam, *Eur. Phys. J. C* **57**, 579 (2008).
- [13] S. Prelovsek and D. Mohler, *Phys. Rev. D* **79**, 014503 (2009).
- [14] S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K.-F. Liu, N. Mathur, and D. Mohler, *Phys. Rev. D* **82**, 094507 (2010).
- [15] C. Alexandrou, J. O. Daldrop, M. Dalla Brida, M. Gravina, L. Scorzato, C. Urbach, and M. Wagner, *J. High Energy Phys.* **04** (2013) 137.
- [16] M. Peardon, J. Bulava, J. Foley, C. Morningstar, J. Dudek, R. Edwards, B. Joó, H.-W. Lin, D. Richards, and K. Juge (Hadron Spectrum Collaboration), *Phys. Rev. D* **80**, 054506 (2009).
- [17] G. S. Bali, S. Collins, and C. Ehmman, *Phys. Rev. D* **84**, 094506 (2011).
- [18] S. Prelovsek and L. Leskovec, [arXiv:1307.5172](https://arxiv.org/abs/1307.5172).
- [19] C. B. Lang, L. Leskovec, D. Mohler, and S. Prelovsek, *Phys. Rev. D* **86**, 054508 (2012).
- [20] J. J. Dudek, R. G. Edwards, and C. E. Thomas, *Phys. Rev. D* **87**, 034505 (2013).
- [21] S. Prelovsek, L. Leskovec, C. B. Lang, and D. Mohler, *Phys. Rev. D* **88**, 054508 (2013).
- [22] G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974).
- [23] E. Witten, *Nucl. Phys.* **B160**, 57 (1979).
- [24] S. Weinberg, *Phys. Rev. Lett.* **110**, 261601 (2013).
- [25] M. Knecht and S. Peris, *Phys. Rev. D* **88**, 036016 (2013).
- [26] J. R. Peláez, J. Nebreda, and G. Ríos, *Prog. Theor. Phys. Suppl.* **186**, 113 (2010).
- [27] Z. Fu, *Phys. Rev. D* **87**, 074501 (2013).
- [28] S. R. Beane, K. Orginos, and M. J. Savage, *Int. J. Mod. Phys. E* **17**, 1157 (2008).
- [29] S. Weinberg, *Physica (Amsterdam)* **96A**, 327 (1979).
- [30] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984); *Nucl. Phys.* **B250**, 465 (1985).
- [31] J. A. Oller and E. Oset, *Nucl. Phys.* **A620**, 438 (1997); **A652**, 407(E) (1999).
- [32] J. A. Oller, E. Oset, and J. R. Peláez, *Phys. Rev. Lett.* **80**, 3452 (1998); *Phys. Rev. D* **59**, 074001 (1999); **60**, 099906(E) (1999); **75**, 099903(E) (2007).
- [33] J. A. Oller and E. Oset, *Phys. Rev. D* **60**, 074023 (1999).
- [34] J. A. Oller and U.-G. Meißner, *Phys. Lett. B* **500**, 263 (2001).
- [35] A. Gómez Nicola and J. R. Peláez, *Phys. Rev. D* **65**, 054009 (2002).
- [36] C. W. Bernard and M. F. L. Golterman, *Phys. Rev. D* **46**, 853 (1992); **49**, 486 (1994).
- [37] S. R. Sharpe and N. Shores, *Phys. Rev. D* **62**, 094503 (2000); **64**, 114510 (2001).
- [38] S. R. Sharpe, [arXiv:hep-lat/0607016](https://arxiv.org/abs/hep-lat/0607016).
- [39] M. Golterman, [arXiv:0912.4042](https://arxiv.org/abs/0912.4042).
- [40] B. C. Tiburzi, *Phys. Rev. D* **79**, 077501 (2009).
- [41] M. Della Morte and A. Jüttner, *J. High Energy Phys.* **11** (2010) 154.
- [42] A. Jüttner, *J. High Energy Phys.* **01** (2012) 007.
- [43] F.-K. Guo, R.-G. Ping, P.-N. Shen, H.-C. Chiang, and B.-S. Zou, *Nucl. Phys.* **A773**, 78 (2006).